NATIONAL INSTITUTE OF TECHNOLOGY KARNATAKA
Department of Mathematical and Computational Sciences

Ph.D. Admission, Dec'14 | Time : 1:30 hours
Date : 15-12-2014 | **Written Aptitude Test** | Max Marks : 30

Instructions:

- Answer any number of questions.
- Answer sheet will be evaluated for a maximum of 30 marks.
- Write all answers on the separate pages.

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1. Let G be a non-trivial group with no non-trivial automorphism. Find the order of G? Justify your answer. [3]

2. The system of equations $Ax = b$ is to be solved iteratively by $\bar{x}_{n+1} = M\bar{x}_n + \bar{b}$. If $A = \begin{bmatrix} 1 & k \\ 2k & 1 \end{bmatrix}$, where $k \neq \sqrt{2}/2$, $k \in \mathbb{R}$, then find a necessary and sufficient condition on k for the convergence of the Jacobi method. [3]

3. Let $f : \mathbb{R} \to \mathbb{R}_l$ be defined by $f(x) = x$ where $\mathbb{R}$ is the set of all real numbers with usual topology and $\mathbb{R}_l$-is the set of all real numbers with lower limit topology. Prove or disprove: f is a homeomorphism. [3]

4. Find the particular integral of the differential equation $(D^2 + 2)y = x^2e^{3x}$. [3]

5. Use the residue theorem to evaluate $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$. [3]

6. Whether the set of all sequences whose terms are the digits 0 and 1 is countable? Justify your answer. [3]

7. State Hahn-Banach extension theorem for continuous linear functional on normed spaces. Discuss the case of unique extension. [4]

8. Let A be a 5×5 real symmetric matrix. If the characteristic polynomial of A is $(\lambda - 2)^2(\lambda + 2)^3$, find α and γ such that $A^{-1} = \alpha A + \gamma I$. [4]

9. Define a cut vertex of a graph and prove that a connected graph with p vertices can have at most $p - 2$ cut vertices. [4]

10. Assume that you are the address administrator at an ISP (Internet Service Provider). You have a 128.20.224.0/20 address block. You have two customers with networks of size 1000 nodes each; two customers whose networks have 500 nodes each; and three customers whose networks have 250 nodes each. What are the address blocks you will assign to these customers? Use notation similar to 128.20.224.0/20 to denote the address blocks you allocate. [5]

11. Assume that passwords are limited to the use of the 95 printable ASCII characters and that all passwords are 10 characters in length. Assume a password cracker with an encryption rate of 6.4 million encryptions per second. How many years will it take to test exhaustively all possible passwords on a UNIX system? [5]

12. What is the expected output of the following program (Explain): [5]

```
int f(int k)
{
int F;
if(k < 3) F=k;
else F=f(k-1)*f(k-2)+f(k-3);
return F;
}
 int main()
{
printf("The value is: %d",f(4));
}
```

13. Consider the five letters with percentage of occurrence in the text as A: 35%, B: 20%, C: 20%, D: 15% and E: 10 %. If a binary tree is generated using Huffman-code algorithm, with assigning 0 to every left edge and 1 to every right edge, then generate the codes for each alphabets/symbols. [5]

14. (a) Draw the binary search tree with six nodes, whose pre order traversal of the keys produces the sequence $\langle 12, 10, 11, 15, 13, 14 \rangle$. [2]

    (b) Suppose that you have 3 algorithms whose running times are listed below. (Assume that these are the exact number of operations performed as a function of the input size n.) Suppose you have a computer that can perform 10^{10} operations per second and you need to compute the result in at most one hour of computation. For each of the algorithms, what is the largest input size n for which you would be able to get the result within an hour? [3]

    i. n^2
    ii. $100n^2$
    iii. n^3

15. (a) How many topological orderings does the following graph have? [2]

    (b) Suppose that you are given an array with n distinct entries. The array has the unimodal property; that is for some index p between 1 and n, the values in the array entries increase upto position p and then decrease the remainder of the way until position n. Give an $O(\log n)$ time algorithm to find the "peak entry" in the array. [3]

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