NATIONAL INSTITUTE OF TECHNOLOGY KARNATAKA
Department of Mathematical and Computational Sciences

Ph.D. Admission, June'15 Time : 1:30 hours

Date : 18-06-2015 **Written Test** Max Marks : 30

Instructions:

- Answer any number of questions.
- Answer sheet will be evaluated for a maximum of 30 marks.
- Write all answers on the separate pages.

---

1. Let A be the set of all real valued function on $\mathbb{R}$. Define $(f+g)(x) = f(x) + g(x)$ and $(f * g)(x) = f(g(x))$ for every $x \in \mathbb{R}$. Is $(A, +, *)$ a ring? Justify your answer. [4]

2. Evaluate the integral $\oint_C f(z)dz$ counterclockwise along C for $f(z) = z^2 exp(\frac{1}{z})$, where C is the unit circle $|z| = 1$. [3]

3. What is the minimum and maximum number of possible vertices in a connected graph with 60 edges? Justify your answer. [3]

4. Let $V = \{(x, y, z, u) \in \mathbb{R}^4 : y + z + u = 0\}$ and $W = \{(x, y, z, u) \in \mathbb{R}^4 : x + y = 0, z = 2u\}$ be two subspaces of $\mathbb{R}^4$. Find the dimensions of the subspaces $V + W$ and $V \cap W$. Also find a basis for $V \cap W$. [4]

5. Determine the number of iterations necessary to solve $f(x) = x^3 + 4x^2 - 10 = 0$ using bisection method to obtain the accuracy 10^{-3} when $a_0 = 1$ and $b_0 = 2$. [3]

6. Consider the initial value problem (IVP) $y' = \sqrt{|y|}$, $y(0) = 0$. Is the solution of the IVP unique? Justify. [3]

7. A map f between normed spaces X and Y is said to be *open* if the image (under f) of every open set in X is open in Y. Let $T : X \to Y$ be a non-zero linear map. [4]

   (✠) : `If` T `is an open mapping, then` T `is onto (surjective).`

   (a) Prove the statement (✠).

   (b) Under what additional assumptions on X and Y, is the converse of (✠) true?

   (c) Suppose that Y is either the set of real or complex numbers and X is any normed space. Is the converse of (✠) true? Justify.

8. Prove that every compact Hausdorff space is a normal space. [3]

9. Prove that every totally bounded metric space is separable. [3]

10. (a) Consider the following function in C: 5

```
int f(int n)
{
  if (n<=1) return 1;
  n= (n-1)*(n-1)-2-n*n+3*n;
  f(n);
  printf(%d, n);
}
```

What is the output, if the initial call is f(6)? Explain.

(b) Consider the following piece of "C" code:

```
int x,b,k,n,z=1;
x=b,k=n;
while(k!=0)
{
  if(odd(k))
  z=z*x;
  x=x*x;
  k=k/2;
}
```

When the *while* loop terminates, which of the following is true? Justify the answer.

A. $z = b^n$
B. $z = b^{n-1}$
C. $x = b^n$
D. None of the above.

(c) Write a simplified boolean function (with minimum number of literals) equivalent to the boolean function $(\sim\ (\sim\ p \wedge q) \wedge \sim\ (\sim\ p \wedge \sim q)) \vee (p \wedge r)$

11. (a) What is the infix and post fix format of the prefix expression: $ABC + *DE * /$ 5

(b) Construct a DFA for the language that accepts all the binary strings without a substring 001.

(c) The running time of an algorithm given by

$$T(n) = \begin{cases} 1 & \text{if } n \leq 3 \\ T(n-1) + T(n-2) - T(n-3) & \text{otherwise.} \end{cases}$$

Then $T(n) =$

A. $O(n)$
B. $O(\log n)$
C. $O(n^2)$
D. none of the above.

Justify the answer with proper explanations.

12. Suppose you have algorithms with the following running times listed below. (Assume that these are the exact running times). [5]

    i. n^2 iii. $100n^2$ v. 2^n

    ii. n^3 iv. $n \log_2 n$

    (a) How much slower do each of these algorithms run when you double the input size?

    (b) How much faster do each of these algorithms run when the input size is reduced to one third?

13. Solve the following recurrence equation. Use recursion tree method. [5]

    (a) $T(n) = \begin{cases} 2T(n/2) + n & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$

    (b) $T(n) = \begin{cases} 4T(n/2) + n^3 & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$

14. Consider a hard disk with 16 recording surfaces $(0 - 15)$ having 16384 cylinders (0-16383) and each cylinder contains 64 sectors (0-63). Data storage capacity in each sector is 512 bytes. Data are organized cylinderwise and the addressing format is <cylinder no., surface no., sector no.>. A file of size 42797 KB is stored in the disk and the starting disk location of the file is <1200, 9, 40>. What is the cylinder number of the last sector of the file, if it is stored in a contiguous manner? [5]

15. Given the class based Network ID of 200.1.1.0/24. You are required to establish four subnets with the following number of hosts. NetA 72, NetB 35, NetC 20, NetD 18. List the range of IP addresses in each case. Use slash notation to denote the address blocks you allocate. [5]

⋆⋆⋆⋆⋆⋆⋆